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# Smooth Sliding-Mode Control for Spacecraft Attitude Tracking Maneuvers

Shih-Che Lo* and Yon-Ping Chen†
*National Chiao Tung University, Hsinchu 300, Taiwan, Republic of China*

**A sliding-mode control (SMC) algorithm is derived and applied to quaternion-based spacecraft attitude tracking maneuvers. Based on some interesting properties related to the spacecraft model, a class of linear sliding manifolds is selected. Significantly, a Lyapunov function is introduced in the SMC design, which can avoid the inverse of the inertia matrix and thus simplify the controller design. To improve the transient response before reaching the sliding manifold, the smoothing model-reference sliding-mode control (SMRSMC) is further developed, which requires well-estimated initial conditions. Simulation results are included to demonstrate the usefulness of the SMRSMC method.**

## I. Introduction

THIS paper presents the sliding-mode control (SMC) and its modified version, the smoothing model-reference sliding-mode control (SMRSMC), for quaternion-based robust spacecraft attitude tracking maneuvers.

The SMC is a robust control technique[1] that has been applied to the spacecraft attitude tracking problem by Dwyer and Sira-Ramirez,[2] Dwyer and Kim,[3] and Chen and Lo,[4] using the Rodrigues parameters as the attitude measurement. Recently, most investigators have paid attention to the quaternion-based spacecraft attitude model.[5–7] Here we also design a sliding-mode controller for spacecraft attitude tracking by using the quaternion-based model. However, during the approach to the sliding manifold, the transient response of the system is generally undesirable. To improve such transient response, the SMRSMC algorithm is derived for the spacecraft attitude tracking maneuvers.

Section II presents the kinematic and dynamic equations of a rigid spacecraft. In addition, some interesting properties related to the quaternion-based kinematic equations are exploited. In Sec. III, a class of linear sliding manifolds is introduced and the SMC algorithm is derived for robust attitude tracking maneuvers. Further, the so-called SMRSMC algorithm is presented in Sec. IV. A numerical example of a multiaxial attitude tracking problem is illustrated in Sec. V to verify the SMRSMC control law. Finally, Sec. VI gives the concluding remarks.

## II. Spacecraft Model Description

The general case of a rigid spacecraft rotating under the influence of body-fixed torquing devices is discussed in this section, where the body-fixed control axes do not have to coincide with the principal axes of inertia. The kinematic equation and the dynamic equation are described, respectively, by[8]

$$\begin{cases} \dot{q} = \frac{1}{2}T(Q)\omega \\ \dot{q}_4 = -\frac{1}{2}q^T\omega \end{cases} \tag{1}$$

and

$$J\dot{\omega} = -[\omega\times]J\omega + u + d \tag{2}$$

In the kinematic equation (1), $Q = [q^T \;\; q_4]^T$ denotes the quaternion with $q = [q_1 \;\; q_2 \;\; q_3]^T$, $\omega = [\omega_1 \;\; \omega_2 \;\; \omega_3]^T$ is the angular velocity vector, and

$$T(Q) = (q_4 I_3 + [q\times]) \tag{3}$$

where $I_3$ is a $3 \times 3$ identity matrix and $[q\times]$ is a skew-symmetric matrix expressed by

$$[q\times] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{4}$$

In the dynamic equation (2), $u = [u_1 \;\; u_2 \;\; u_3]^T$ is the control vector, $d = [d_1 \;\; d_2 \;\; d_3]$ represents the bounded disturbances, and $J$ is the inertia matrix with bounded time derivative and uncertainties in its components. Here, the subscripts 1, 2, and 3 denote the three body-fixed control axes. The kinematic equation (1) can be rewritten in a more compact form as[9]

$$\dot{Q} = \tfrac{1}{2}E(Q)\omega \tag{5}$$

where

$$E(Q) = \begin{bmatrix} T(Q) \\ -q^T \end{bmatrix}$$

Note that the elements of $Q$ are constrained by

$$\|Q\| = 1 \quad \text{or} \quad q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \tag{6}$$

and we assume $q_4 \geq 0$ to avoid the sign ambiguity.[8] Two properties related to the spacecraft mathematical model are given next.

**Property 1**

The matrix $T(Q)$, defined in Eq. (3), has the following properties; a) $x^T T(Q)x = q_4\|x\|^2, \forall x \in \Re^3$; b) $T^{-1}(Q) = T^T(Q) + q_4^{-1}[qq^T]$, for $q_4 \neq 0$; and c) $\|T(Q)\|_{i2} = 1$, where $\|\cdot\|$ and $\|\cdot\|_{i2}$, respectively, denote the two norm and the induced two norm.

Property b has been presented by Dwyer,[5] and it is provided only for $q_4 \neq 0$. As a consequence, the tracking control developed in the following sections will work only under the condition $q_4 \neq 0$.

**Property 2**

The matrix $E(Q)$ has the following properties: a) $E^T(Q)E(Q) = I_3$ and $\|E(Q)\|_{i2} = 1$; b) $(d/dt)[E^T(Q)\dot{Q}] = E^T(Q)\ddot{Q}$; and c) $\|\omega\| = 2\|\dot{Q}\|$. By using properties a and b, from Eq. (5) the angular velocity $\omega$ and its time derivative can be expressed as

$$\begin{cases} \omega = 2E^T(Q)\dot{Q} \\ \dot{\omega} = 2E^T(Q)\ddot{Q} \end{cases} \tag{7}$$

All of these properties will apply to the sliding-mode controller design.

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*Graduate Student, Department of Control Engineering.

†Associate Professor, Department of Control Engineering.

## III. Attitude Tracking by SMC

Here, a sliding-mode controller is developed to achieve the robust attitude tracking. To avoid the singularity of $T(Q)$ that occurs as $q_4 = 0$, let the spacecraft's attitude be restricted in the workspace $W$ defined by[8]

$$W = \left\{ Q \mid Q = \left[ q^T \;\; q_4 \right]^T, \quad \|q\| \le \beta < 1, \quad q_4 \ge \sqrt{1-\beta^2} > 0 \right\} \tag{8}$$

which implies that the range of the Euler rotation angle is limited between $\pm\cos^{-1}[\surd(1-\beta^2)]$. The objective of attitude tracking is that, given desired signals $Q_d$, $\dot{Q}_d$, and $\ddot{Q}_d$ with $\|Q_d\| = 1$, the controller is designed such that the quaternion $Q$ approaches $Q_d$ in the presence of external disturbances and system variations. From Eq. (7), the desired angular velocity and its time derivative can be obtained as

$$\begin{cases} \omega_d = 2E^T(Q_d)\dot{Q}_d \\ \dot{\omega}_d = 2E^T(Q_d)\ddot{Q}_d \end{cases} \tag{9}$$

Because of the redundancy of the quaternion (6), it is sufficient to design the controller such that $q$ traces $q_d$, where $q_d$ denotes the reduced quaternion of $Q_d$. The design procedure of SMC generally contains two fundamental steps as follows.

**Step 1**

Choose the sliding manifold such that on it the goal of control is achieved.

A class of linear sliding vectors are chosen as follows:

$$s = \omega_e + Kq_e \tag{10}$$

where $K$ is a $3 \times 3$ symmetric positive-definite constant matrix, $\omega_e = \omega - v_d$, $q_e = q - q_d$, and $v_d = 2T^{-1}(Q)\dot{q}_d$. It is noticed that $v_d$ is different from $\omega_d$ defined in Eq. (9). In Vadali's paper,[6] a similar sliding vector was introduced, but it was applied to the attitude regulation problem. Suppose the system invariantly stays on the linear sliding manifold $s = 0$. Taking a nonlinear transformation $\frac{1}{2}T(Q)$ on Eq. (10) yields

$$\dot{q}_c + \tfrac{1}{2}T(Q)Kq_e = 0 \tag{11}$$

Consider a candidate of Lyapunov function

$$V_e = \tfrac{1}{2}q_e^T K q_e \ge 0 \tag{12}$$

From property 1a, the time derivative of $V_e$ is

$$\dot{V}_c = q_e^T K \dot{q}_e = -(q_4/2)q_e^T K^2 q_e \le 0 \tag{13}$$

Since $q_4 > 0$ in the workspace $W$ and $K$ is positive definite, the equality in Eq. (13) holds only when $q_e = 0$. Hence, $V_e$ is really a Lyapunov function, and thus the tracking errors $q_e$ will become zero as $t \to \infty$. Besides, from Eqs. (12) and (13), we have

$$\dot{V} = (\dot{V}_e/V_e)V_e \le -(q_4 \cdot \lambda_m)V_e \le -\left[\sqrt{1-\beta^2}\cdot\lambda_m\right]V_e \tag{14}$$

where $\lambda_m$ is the minimum eigenvalue of $K$. As a result, the exponential convergence rate of $q_e$ is not less than $[\surd(1-\beta^2)\cdot\lambda_m]/2$. The details and similar results have been shown in Chen and Lo.[4]

**Step 2**

Design the control law such that the reaching and sliding on the sliding manifold are satisfied

To avoid the inverse of the inertia matrix, a candidate of Lyapunov function is introduced as

$$V_r = \tfrac{1}{2}s^T J s \ge 0 \tag{15}$$

Using Eqs. (1), (2), and (10), we have

$$\dot{V}_r = s^T\{-[\omega\times]J\omega + u + d - J\dot{v}_d + JK(\dot{q} - \dot{q}_d) + \dot{J}s\} \tag{16}$$

Assuming $\|\dot{J}\|_{i2} \le \sigma_{\dot{J}}$ and letting $J = J_0 + \Delta J$ where $J_0$ and $\Delta J$ denote the nominal and uncertain part of the inertia matrix with $\|\Delta J\|_{i2} \le \sigma_J$, we set the SMC law as

$$u = [\omega\times]J_0\omega + J_0\dot{v}_d - J_0K\left[\tfrac{1}{2}T(Q)\omega - \dot{q}_d\right] + \tau \tag{17}$$

where from property 1b,

$$\dot{v}_d(Q,\omega,\dot{q}_d,\ddot{q}_d) = 2T^{-1}(Q)\ddot{q}_d - T^{-1}(Q)T(\dot{Q})v_d$$

and $\tau = [\tau_1 \;\; \tau_2 \;\; \tau_3]^T$ with

$$\tau_i = -g_i(Q,\omega,q_d,\dot{q}_d,\ddot{q}_d)\cdot\mathrm{sgn}(s_i), \qquad i = 1,2,3 \tag{18}$$

Therefore, Eq. (16) becomes $\dot{V}_r = s^T[\Psi + \tau] = \sum_{i=1}^3 s_i[\Psi_i + \tau_i]$, where $\Psi_i$ is the $i$th component of $\Psi = \{-[\omega\times]\Delta J\omega - \Delta J\dot{v}_d + \Delta JK[\frac{1}{2}T(Q)\omega - \dot{q}_d] + d + \dot{J}s\}$. Because $\Delta J$ and $\dot{J}$ are bounded, it can be found that $\|\Psi_i\| < \Psi_i^{\max}(Q,\omega,q_d,\dot{q}_d,\ddot{q}_d)$. Obviously, if the gains $g_i$ in Eq. (18) are chosen as

$$g_i(Q,\omega,q_d,\dot{q}_d,\ddot{q}_d) \ge \Psi_i^{\max}(Q,\omega,q_d,\dot{q}_d,\ddot{q}_d) \tag{19}$$

then $\dot{V}_r \le -\sum_{i=1}^3 |s_i|[g_i(Q,\omega,q_d,\dot{q}_d,\ddot{q}_d) - \Psi_i\cdot\mathrm{sgn}(s_i)] \le 0$, where the equality is true only for $s = 0$. This guarantees the reaching and sliding on the sliding manifold $s = 0$.

Although the control law (17) for attitude tracking is developed, the system transient performance is usually unpredictable and undesirable before reaching the sliding manifold. To eliminate such drawbacks, a modified scheme, called the SMRSMC, is developed in the next section.

## IV. SMRSMC

Figure 1 shows the block diagram of the SMRSMC scheme. The block of the smoothing reference model (SRM) contains two sets of inputs, the desired attitudes $\{Q_d, \dot{Q}_d, \ddot{Q}_d\}$ and the initial states $\{Q(t_0^-), \omega(t_0^-)\}$. These initial states are assumed to be well estimated such that $\|Q(t_0^-) - Q(t_0)\| \le \alpha$ and $\|\omega(t_0^-) - \omega(t_0)\| \le \alpha$, where $\alpha$ is a small positive value and $Q(t_0)$ and $\omega(t_0)$ represent the real initial states of the spacecraft. The outputs of SRM, $\{Q_c, \omega_c, \dot{\omega}_c\}$, are used as command signals of the sliding-mode controller. The main function of the SRM is to generate $\{Q_c, \omega_c, \dot{\omega}_c\}$ satisfying

$$\dot{Q}_c = \tfrac{1}{2}E(Q_c)\omega_c \tag{20}$$

In addition, $Q_c$ has to track the desired $Q_d$ with initial values of $Q_c$ and $\omega_c$ set as

$$Q_c(t_0) = Q\left(t_0^-\right) \quad \text{and} \quad \omega_c(t_0) = \omega\left(t_0^-\right) \tag{21}$$

Note that the choice of SRM is not unique. Here, we adopt the following simple dynamic SRM:

$$\ddot{q}_e + 2\xi\nu\dot{q}_e + \nu^2 q_e = 0 \tag{22}$$

where $\xi \ge 1$, $\nu > 0$, and initial conditions

$$q_e(t_0) = q_c(t_0) - q_d(t_0), \qquad \dot{q}_e(t_0) = \dot{q}_c(t_0) - \dot{q}_d(t_0)$$
$$\dot{q}_c(t_0) = \tfrac{1}{2}T[Q_c(t_0)]\omega_c(t_0) \tag{23}$$

Thus the outputs of SRM $\{Q_c, \omega_c, \dot{\omega}_c\}$ become

$$\begin{cases} q_c = q_d + q_e \\ q_{4c} = \sqrt{1 - q_c^T q_c} \end{cases}, \qquad \begin{cases} \dot{q}_c = \dot{q}_d + \dot{q}_e \\ \dot{q}_{4c} = (-1/q_{4c})q_c^T\dot{q}_c \end{cases}$$
$$\begin{cases} \ddot{q}_c = \ddot{q}_d + \ddot{q}_e \\ \ddot{q}_{4c} = (-1/q_{4c})\left(\dot{Q}_c^T\dot{Q}_c + q_c^T\ddot{q}_c\right) \end{cases} \tag{24}$$
$$\omega_c = 2E^T(Q_c)\dot{Q}_c, \qquad \dot{\omega}_c = 2E^T(Q_c)\ddot{Q}_c$$

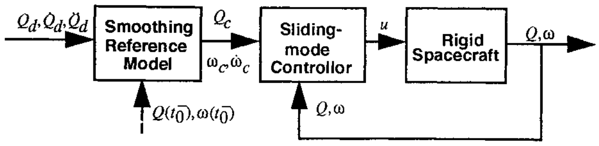


**Fig. 1 Block diagram of the SMRSMC scheme.**

Similar to the previous design procedure of SMC, the SMRSMC controller is developed next.

**Step 1**

The sliding vector is chosen as

$$s = \omega_{ce} + \lambda q_{ce} \tag{25}$$

where $s = [s_1\ s_2\ s_3]^T$, $\omega_{ce} = \omega - \omega_c$, $q_{ce} = q - q_c$, and $\lambda$ is a positive scalar. Next, the stability on the sliding manifold $s = 0$ is discussed. By using the nonlinear transformation $\frac{1}{2}T(Q)$ on Eq. (25) and from Eq. (5), we obtain

$$\dot{q}_{ce} + (\lambda/2)T(Q)q_{ce} = f(\omega_c, Q_{ce}) \tag{26}$$

where $f(\omega_c, Q_{ce}) = \frac{1}{2}\{[q_{ce}\times] + q_{4ce}I_3\}\omega_c$. Let the Lyapunov function be $V_{ce} = \frac{1}{2}q_{ce}^T q_{ce}$, then from Eq. (26) and property 1a, we obtain $\dot{V}_{ce} = -(\lambda q_1/2)\|q_{ce}\|^2 + (q_{4ce}/2)q_{ce}^T\omega_c$. Now, we briefly show that

$$\left|\frac{q_{4ce}}{2}q_{ce}^T\omega_c\right| \le \gamma\|q_{ce}(t)\|^2 \tag{27}$$

where

$$\gamma = \sqrt{\frac{1+\beta}{1-\beta}}\left[\sup_{t\ge t_0}\|\dot{Q}_c(t)\|\right]$$

Since $\|Q\| = \|Q_c\| = 1$, it can be attained that $(q^T + q_c^T)q_{ce} + (q_4 + q_{4c})q_{4ce} = 0$ and then $|q_{4ce}| \le \sqrt{[(1+\beta)/(1-\beta)]}\|q_{ce}\|$. Besides, similar to property 2c, we obtain $\|\omega_c(t)\| = 2\|\dot{Q}_c(t)\|$. Hence, $(q_{4ce}/2)q_{ce}^T\omega_c \le \gamma\|q_{ce}\|^2$, which proves Eq. (27). As to the upper bound of $\|\dot{Q}_c(t)\|$, it is determined by $\sup_{t\ge t_0}\|\dot{Q}_c(t)\| \le (1 + c^{-1})[\sup_{t\ge t_0}\|\dot{q}_d(t)\| + \sup_{t\ge t_0}\|\dot{q}_e(t)\|]$, where $c = \inf_{t\ge t_0} q_{4d}(t) + q_{4e}(t) > 0$. Thus $\dot{V}_{ce}(t) \le [-(q_4\cdot\lambda/2) + \lambda]2V_{ce}$. Evidently, if $\lambda$ is appropriately selected such that $\{-[\sqrt{(1-\beta^2)}\cdot\lambda]/2 + \lambda\} < 0$, or $\lambda > 2\gamma/\sqrt{(1-\beta^2)}$, then the stability of the tracking system is guaranteed.

**Step 2**

Select the Lyapunov function $V_s = \frac{1}{2}s^T Js$, and then

$$\dot{V}_s = s^T\left(-[\omega\times]J\omega + u + d - J\dot{\omega}_c + \lambda J\dot{q}_{ce} + \tfrac{1}{2}\dot{J}s\right)$$

If the control law is chosen as

$$u = u_f(t) + \tau(Q,\omega) = [\omega_c\times]J_0\omega_c + J_0\dot{\omega}_c + \tau(Q,\omega) \tag{28}$$

where $\tau = [\tau_1\ \tau_2\ \tau_3]^T$ with $\tau_i = g_i\cdot\mathrm{sgn}(s_i)$, and

$$g_i > \left|\left\{[\omega_c\times]J_0\omega_c - [\omega\times]J\omega - \Delta J\dot{\omega}_c + \lambda J\dot{q}_{ce} + \tfrac{1}{2}\dot{J}s + d\right\}_i\right| \tag{29}$$

for all $Q \in W$, then $\dot{V}_s < 0$, which guarantees the reaching and sliding on the sliding manifold. The spacecraft attitude tracking is fulfilled.

Because of the existence of nonideality in the practical implementation of sgn $(s_i)$, the control law $\tau$ in Eq. (28) generally suffers from the chattering problem. To alleviate such undesirable performance, several methods have been presented.[3,10] Here, the sign function is modified as[10]

$$\mathrm{sat}(s_i,\epsilon) = \begin{cases} 1 & s_i > \epsilon \\ s_i/\epsilon & |s_i| \le \epsilon \\ -1 & s_i < -\epsilon \end{cases} \tag{30}$$

The thickness $\epsilon$ of the sliding layer $|s_i| \le \epsilon$ is selected considering the hardware capability and the reduction of accuracy and robustness.[10]

The benefits of the SMRSMC are as follows. First, since the initial conditions of SRM are set as Eq. (21), from Eq. (25), we have $\|s(t_0)\| \le (1+\lambda)\alpha$, the system is initially placed near the sliding manifold, thus to improve the undesired transient response. In addition, the use of high control gain during a long transient period is avoided. Since the command signals $Q_c$ and $\omega_c$ smoothly interpolate the desired signals and the initial conditions $\|s\|$, $\|q_{ce}\|$, $\|\dot{q}_{ce}\|$, and $\|\omega_{ce}\|$ always remain small. As a result, the maximum values of $g_i$ can be evaluated as constant values.

## V. Multiaxial Attitude Tracking Maneuvers

Here we present an example with numerical simulation results to demonstrate the developed SMRSMC method. The nominal part $J_0$ and the uncertain part $\Delta J$ of the inertia matrix are

$$J_0 = \begin{bmatrix} 1200 & 0 & 0 \\ 0 & 2200 & 0 \\ 0 & 0 & 3100 \end{bmatrix}$$

and

$$\Delta J = \begin{bmatrix} 0 & 100 & -200 \\ 100 & 0 & 300 \\ -200 & 300 & 0 \end{bmatrix}$$

The initial conditions are set as $Q(0) = [0, 0.5, 0.5, 0.7071]^T$ and $\omega(0) = [-0.0005, 0.0008, 0.001]^T$, and the measured initial conditions are $Q(0^-) = [0.001, 0.49, 0.51, 0.707]^T$ and $\omega(0^-) = [0, 0, 0]^T$. Besides, the workspace $W$ of the spacecraft is defined by $\beta^2 = 0.75$ and the external disturbances are injected by $d_i(t) = 0.5\cdot\sin(t)(\mathrm{Nt}\cdot\mathrm{m})$ for $i = 1, 2, 3$. Based on this model, the desired multiaxial attitude tracking maneuvers are

$$q_d(t) = \begin{bmatrix} 0.5\cdot\cos[(\pi/50)t] \\ 0.5\cdot\sin[(\pi/50)t] \\ -0.5\cdot\sin[(\pi/50)t] \end{bmatrix}$$

and the attitude tracking mission requirements are set as the settling time 50 s and $|u_i| \le 100\,(\mathrm{N}\cdot\mathrm{m})$ for $i = 1, 2, 3$. To satisfy these specifications, the SMRSMC scheme employs the SRM shown in Eqs. (22–24) with $\xi = 1$ and $\nu = 0.12$. The positive scalar $\lambda$ of the sliding manifold (10) and (25) is selected to be $\lambda = 0.2$. The gains in the control law (28) are chosen as $g_i = 40$ for $i = 1, 2, 3$. To ameliorate the chattering, the sign function is replaced by the saturation function (30) with $\epsilon = 0.01$. Furthermore, to highlight the features of the SMRSMC scheme, the SMC scheme presented in Sec. III is also applied to the preceding tracking maneuvers, where the sliding manifold is chosen as Eq. (10) with $K = \lambda\cdot I^3$, and the control law (17) employs the gains in Eq. (18) as

$$g_i = \sigma_J\left(\|\omega\|^2 + \|\dot{v}_d\| + \lambda\|\dot{q}_e\|\right) + 1, \qquad \text{for } i = 1, 2, 3$$

Simulation results on the attitude tracking maneuvers are shown in Figs. 2–7. In accordance with the sliding-mode theory, Fig. 2

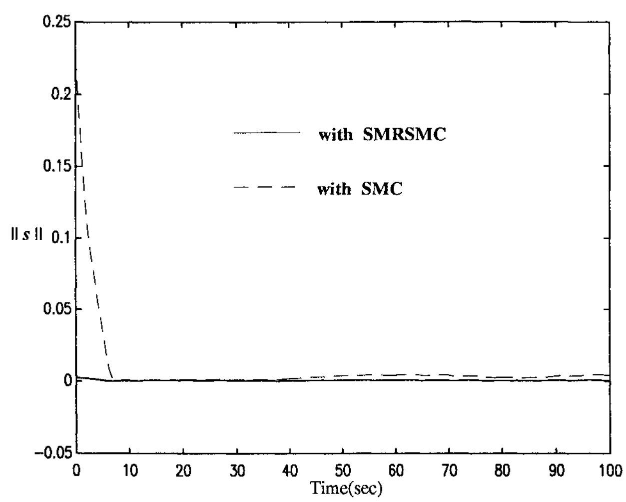


**Fig. 2 Two norm of the sliding vectors of the SMC and SMRSMC schemes.**

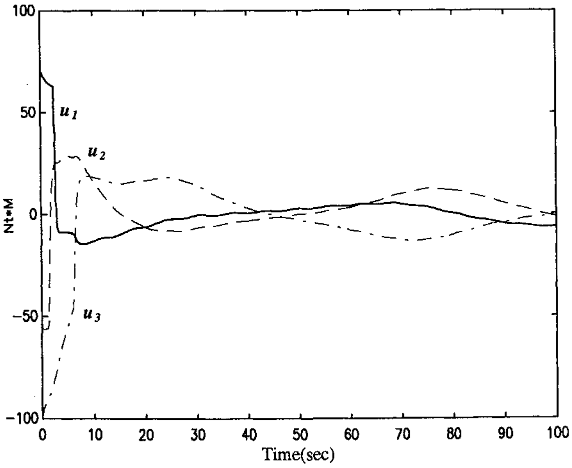


**Fig. 3 Control torques of the SMC scheme.**

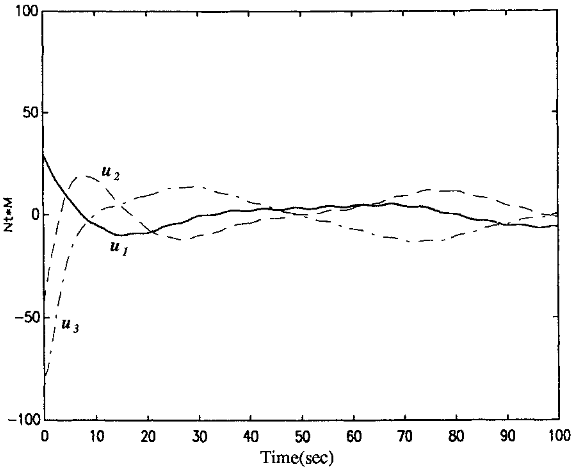


**Fig. 4 Control torques of the SMRSMC scheme.**

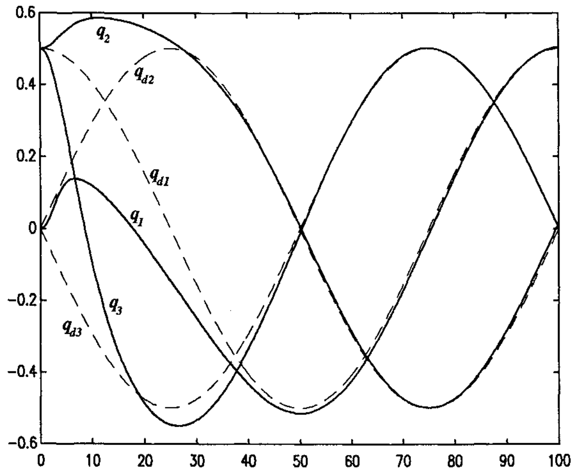


**Fig. 5 Attitude tracking response of the SMC scheme.**

demonstrates that the sliding vectors are driven to the sliding layer and then constrained therein. Most significantly, Fig. 2 also demonstrates the improvement of transient behavior by using the SMRSMC technique before reaching the sliding mode. All of the control torques presented in Figs. 3 and 4 satisfy the mission requirements and possess no chattering; however, Fig. 3 shows that the SMC scheme requires larger torques with a faster change rate, which sometimes makes the use of SMC algorithms difficult or even impractical. Therefore, the SMRSMC scheme should be adopted for

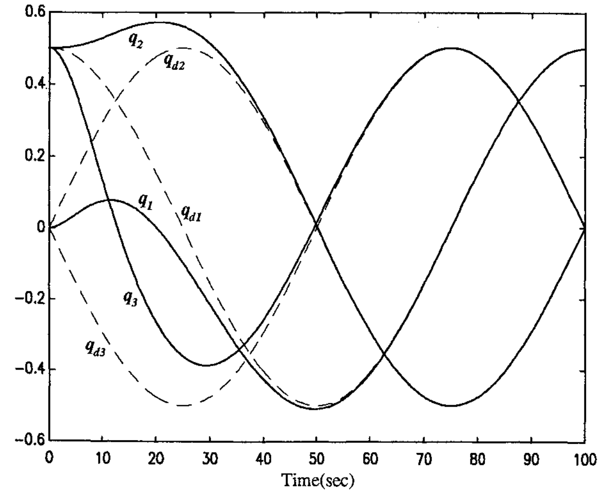


**Fig. 6 Attitude tracking response of the SMRSMC scheme.**

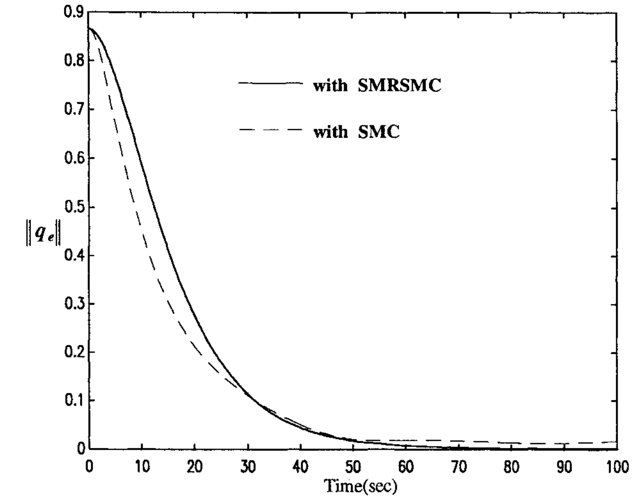


**Fig. 7 Two norm of attitude tracking errors of the SMRSMC and SMC schemes.**

generating useful, reasonable control torques as depicted in Fig. 4. The successful trace of the quaternions to the desired signals by using the SMC and the SMRSMC schemes can be directly seen from Figs. 5 and 6. Hence, the robustness to parameter variations $\Delta J$ and external disturbances $d(t)$ is verified. Finally, Fig. 7 shows the two norm of the attitude tracking errors. In view of these simulation results, the superb performance of the SMRSMC scheme is evident.

## VI. Conclusions

The SMRSMC scheme has been successfully applied to the spacecraft attitude tracking maneuvers. Furthermore, it improves the undesirable transient response induced in the conventional sliding-mode controller. A class of linear sliding manifold and two significant Lyapunov functions are introduced in both the controller design and system stability analysis. Besides, the convergence rate of the tracking errors can be determined by suitably choosing the linear sliding manifold. To deal with the chattering problem, the sign function is replaced by the saturation function. Finally, an example of spacecraft multiaxial attitude tracking maneuvering is given to verify the usefulness of the smoothing model-reference sliding-mode controller.

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